Exercise 48

Calculate y'.

$$y = x \tanh^{-1} \sqrt{x}$$

Solution

Calculate y' by using the chain and product rules.

$$\begin{aligned}
y' &= \frac{d}{dx}\left(x \tanh^{-1} \sqrt{x}\right) \\
&= \left[\frac{d}{dx}(x)\right] \tanh^{-1} \sqrt{x} + x \left[\frac{d}{dx}(\tanh^{-1} \sqrt{x})\right] \\
&= (1) \tanh^{-1} \sqrt{x} + x \left[\frac{1}{1-(\sqrt{x})^2} \cdot \frac{d}{dx}(\sqrt{x})\right] \\
&= \tanh^{-1} \sqrt{x} + x \left[\frac{1}{1-x} \cdot \left(\frac{1}{2} x^{-1/2}\right)\right] \\
&= \tanh^{-1} \sqrt{x} + x \left[\frac{1}{1-x} \left(\frac{1}{2\sqrt{x}}\right)\right] \\
&= \tanh^{-1} \sqrt{x} + \frac{\sqrt{x}}{2(1-x)}
\end{aligned}$$